GERRYMANDERING: HIJACKING DEMOCRACY ONE NONCONVEX REGION AT A TIME

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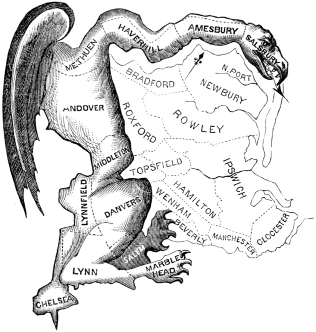
Hamilton College

March 7, 2018



Definition (Gerrymander)

(verb): to manipulate the boundaries of (an electoral constituency) so as to favor one party or class *(Paraphrased from the OED)*

From the American Mathematical Society statement on Gerrymandering:
www.ams.org/about-us/governance/policy-statements/gerrymandering

> *The American Statistical Association (ASA) and American Mathematical Society (AMS) attest to the following facts:*

FACT 1: *Existing requirements for districts generally do not prevent partisan gerrymandering.*

FACT 2: *It has become easier to design district plans that strongly favor a particular partisan outcome.*

FACT 3: *Modern mathematical, statistical, and computing methods can be used to identify district plans that give one of the parties an unfair advantage in elections.*

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…but is has some ideas worth thinking about!

- What is a (Fair) District Plan?

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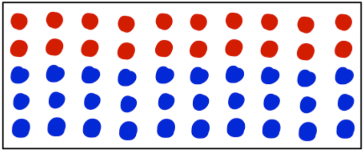
- Who's Who in the Mathematics of Districting

WHAT IS A (FAIR) DISTRICT PLAN?

50 voters across 5 districts recently voted for district representatives (from the Blue or Red party):



30 voters (60%): Blue party;
20 voters (40%): Red party.

The number of Blue and Red representatives depends on districts:

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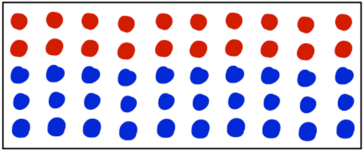

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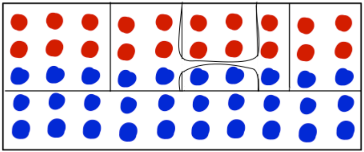

1 Blue Reps
4 Red Reps
"Anything Goes"

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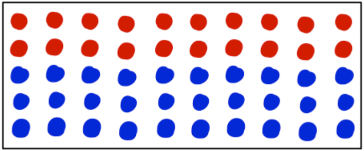

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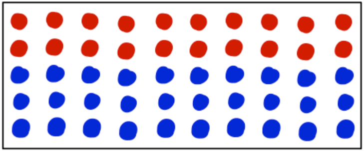

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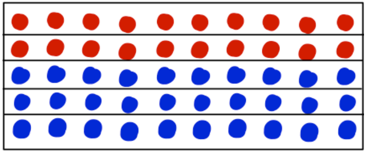

3 Blue Reps
2 Red Reps
"Proportional Representation"

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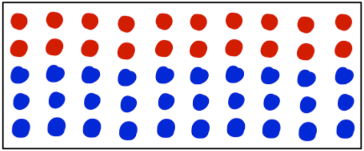

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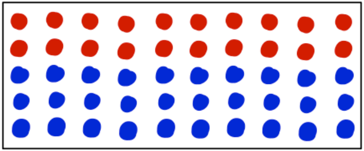

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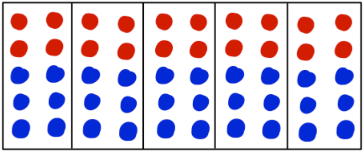

5 Blue Reps
0 Red Reps
“Blue Gerrymander”

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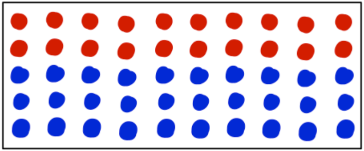

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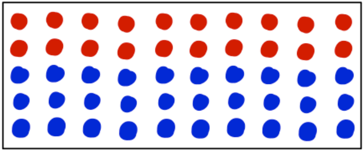

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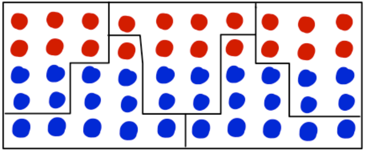
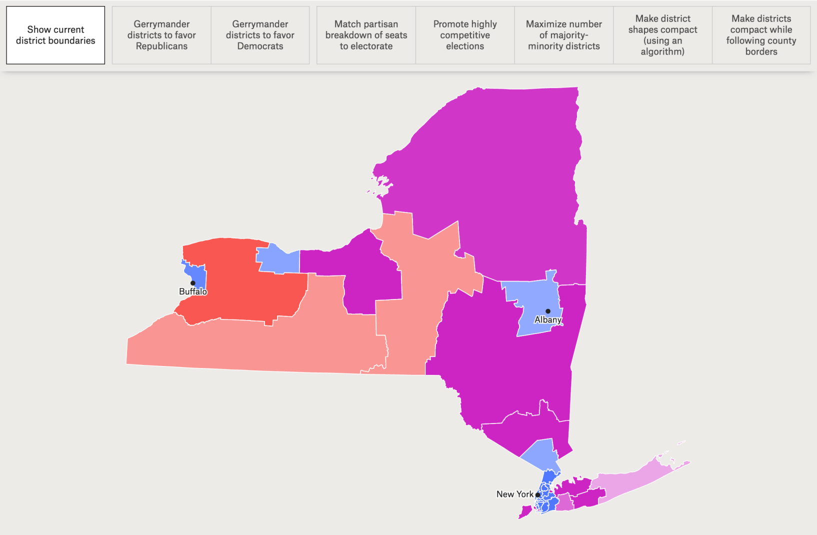


2 Blue Reps
3 Red Reps
"Red Gerrymander"

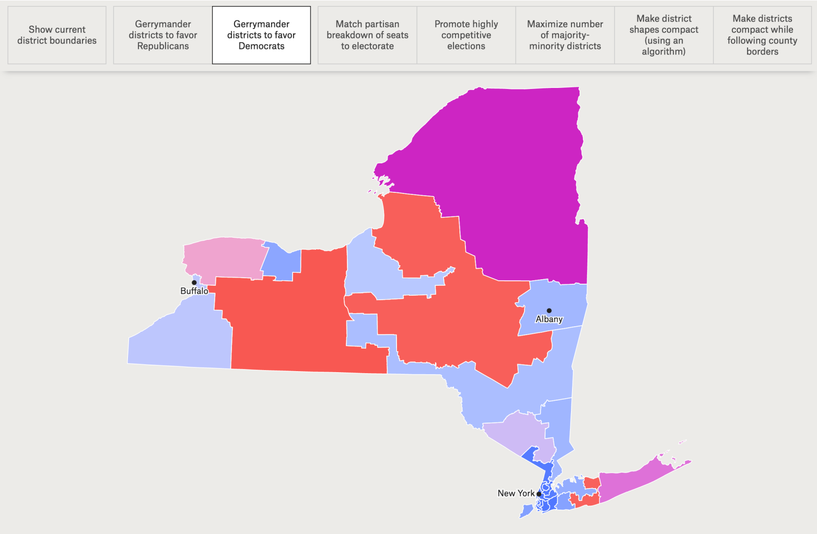
PACKING AND CRACKING (NY EDITION)

Current Congressional Districts:



https://projects.fivethirtyeight.com/redistricting-maps/

A Hypothetical Democratic Gerrymander:



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Federal Government:

> *"The Equal Protection Clause [of the United States Constitution] demands no less than substantially equal state legislative representation for all citizens, of all places as well as of all races."*
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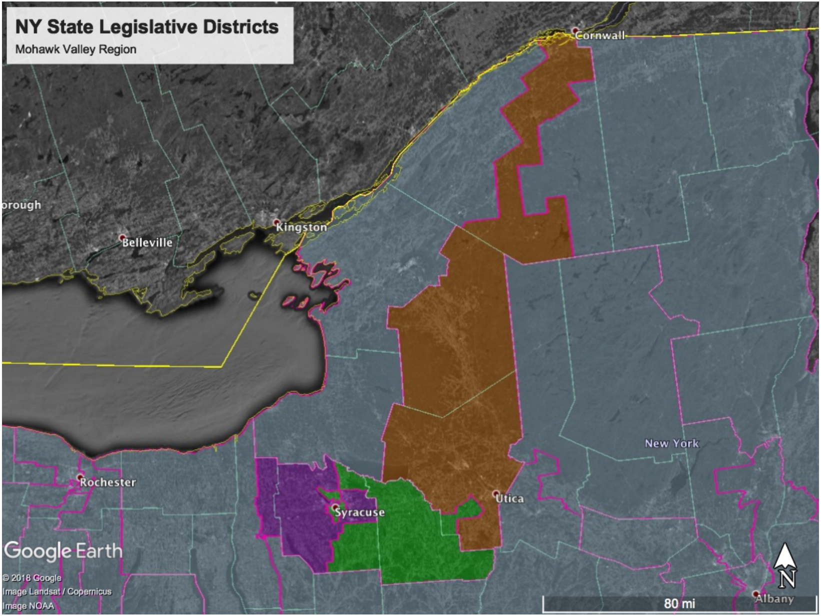
State Criteria:

- Contiguity (no disconnected districts) [NY: all maps]
- Compactness (within reason, residents of districts should live "as close as possible to each other" [NY: congressional maps]
- Community of Interest (common social, economic, or political interests)
- Political Boundaries (no splitting up towns or counties)

NY, I HAVE SOME QUESTIONS...



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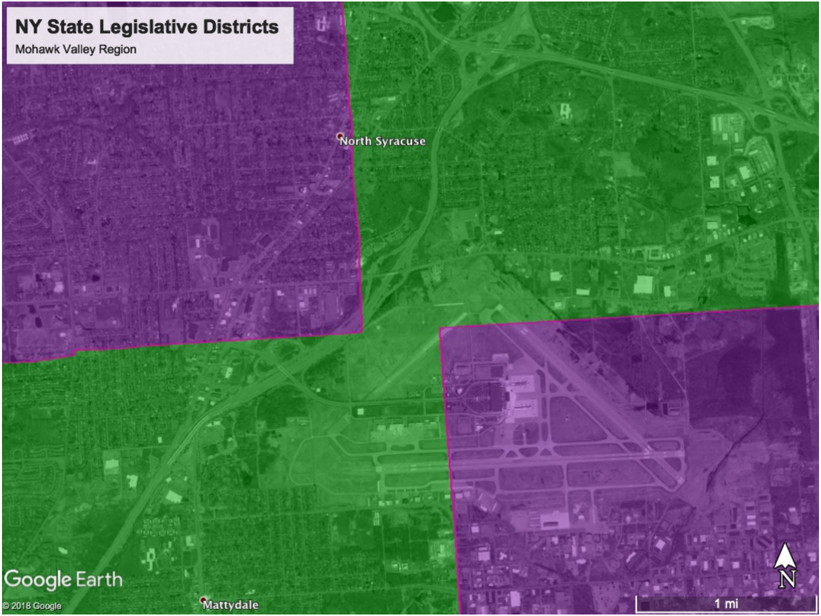

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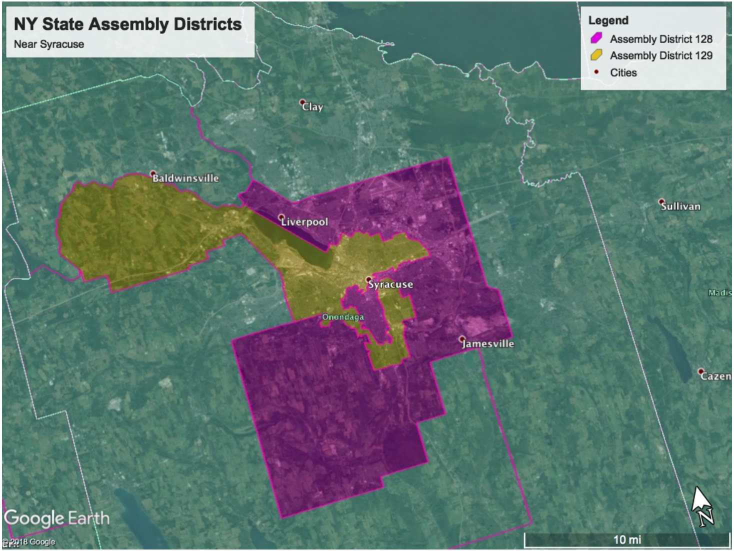
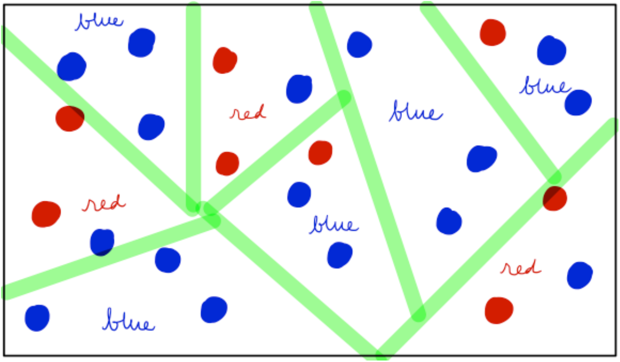


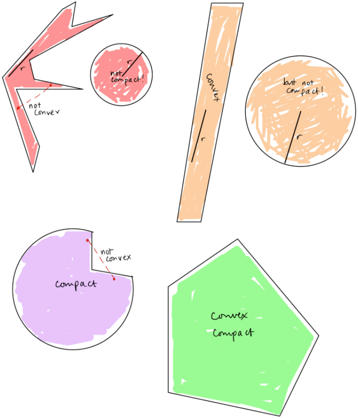
MATHEMATICAL CRITERIA AND METRICS FOR (PARTISAN) FAIRNESS

- Proportionality: does the seat breakdown reflect the voters' general preferences?
- Convexity and Compactness: how weird does the map look?
- Efficiency Gap: how many votes are wasted?
- Partisan Symmetry: who wins if voters change parties?

If 67% of voters across a region prefer the Party X, then it is fair for $\sim$67% of the seats in that region to go to the Party X.



67% Prefer Blue; Blue Wins 62.5%; Seems Pretty Fair

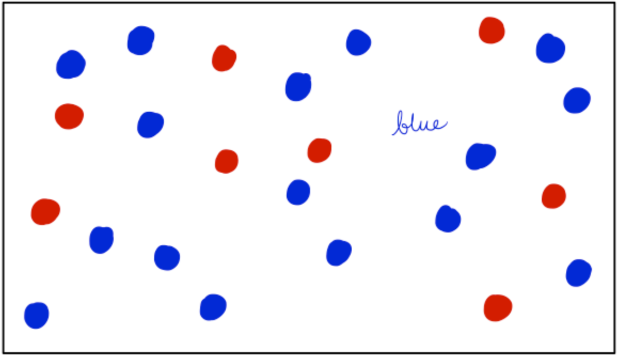


The hope: Convex, compact maps will make it more difficult to gerrymander.

The reality: Unintuitively, it may just make it harder to spot gerrymandering.

Theorem (Ham Sandwich Theorem)

Given two finite sets of points in the plane, blue and red, both with an even number of points and such that no three colored points are collinear, there is a line that simultaneously splits both colors in half.



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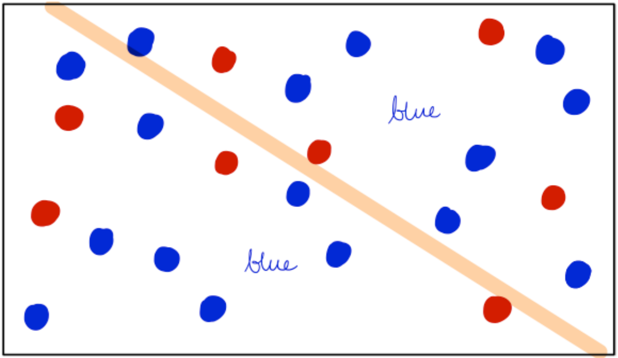

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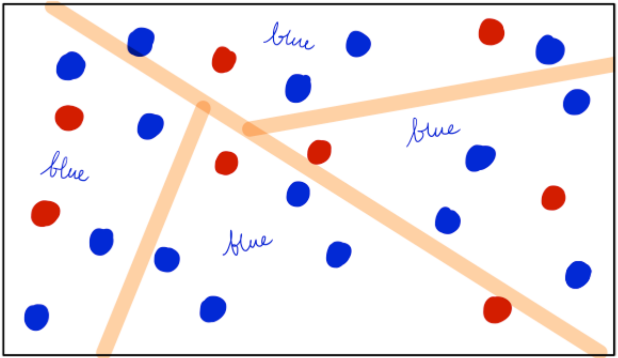

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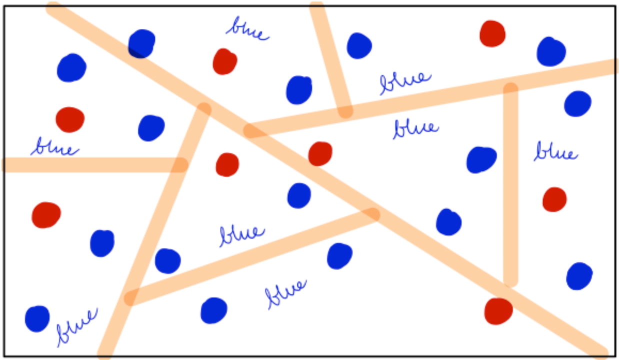

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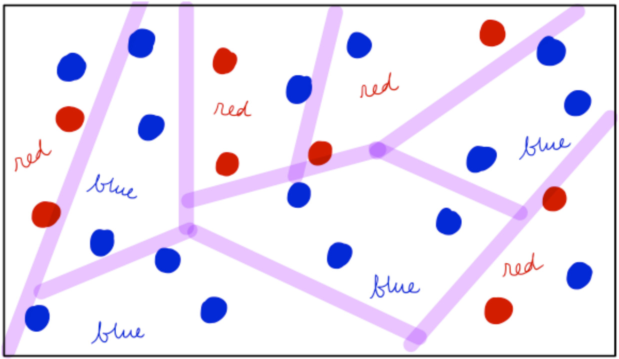
(Variations allow the minority party to gerrymander convexly, too.)



Red wins half!

- Existence theorems guarantee that there are ways to separate a region into (relatively) convex, compact districts.
- High-power computing makes it possible to actually do so.
- Unintuitively, this makes the problem worse: we can’t necessarily eyeball a map to find the problems.

THE EFFICIENCY GAP

Inputs: Vote tallies by district, for each party

- Votes for Blue, Red in the District
- Votes for the District Loser (Lost Votes)
- Votes over 50% for the District Winner (Excess Votes)
- Wasted Votes: Lost + Excess Votes

Output: EG(*District Plan*), a measure of how large the gap in votes wasted between parties.

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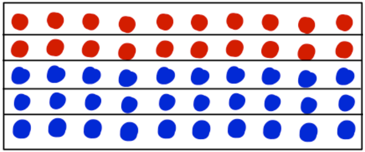
Simplified Efficiency Gap:

Inputs: Vote margins and seats won for a party

Output: sEG(*Party*), a measure of how disadvantaged a party was after the vote

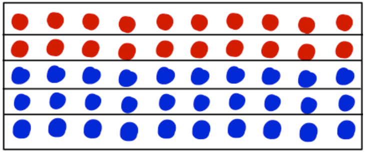
- Positive: the party was advantaged under this plan
- Negative: the party was disadvantaged under this plan



Proportional Representation

	Total Votes		Lost Votes		Excess Votes		Wasted Votes	
Dist.	B	R	B	R	B	R	B	R
D 1	0	10	0	0	0	4	0	4
D 2	0	10	0	0	0	4	0	4
D 3	10	0	0	0	4	0	4	0
D 4	10	0	0	0	4	0	4	0
D 5	10	0	0	0	4	0	4	0
Totals	30	20	0	0	12	8	12	8

$$\text{EG}(\textit{District Plan}) = \frac{12 - 8}{50} = 8\%$$

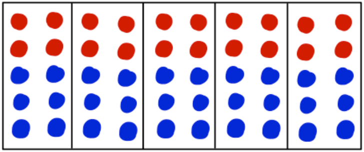


Proportional Representation

Overall Percentages Seats		Overall Percentages Votes	
B	R	B	R
60	40	60	40

$$\text{sEG}(Blue) = (60 - 50) - 2(60 - 50) = -10\%$$

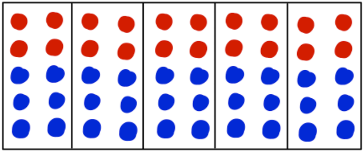
$$\text{sEG}(Red) = (40 - 50) - 2(40 - 50) = +10\%$$



Blue Gerrymander

	Total Votes		Lost Votes		Excess Votes		Wasted Votes	
Dist.	B	R	B	R	B	R	B	R
D 1	6	4	0	4	0	0	0	4
D 2	6	4	0	4	0	0	0	4
D 3	6	4	0	4	0	0	0	4
D 4	6	4	0	4	0	0	0	4
D 5	6	4	0	4	0	0	0	4
Totals	30	20	0	20	0	0	0	20

$$\text{EG}(\textit{District Plan}) = \frac{20 - 0}{50} = 40\%$$

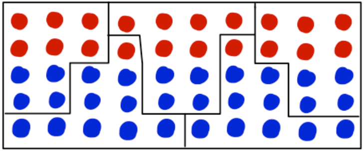


Blue Gerrymander

Overall Percentages Seats		Overall Percentages Votes	
B	R	B	R
100	0	60	40

$$\mathrm{sEG}(\mathit{Blue}) = (100 - 50) - 2(60 - 50) = +30\%$$

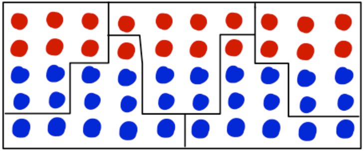
$$\mathrm{sEG}(\mathit{Red}) = (0 - 50) - 2(40 - 50) = -30\%$$



Red Gerrymander

	Total Votes		Lost Votes		Excess Votes		Wasted Votes	
Dist.	B	R	B	R	B	R	B	R
D 1	4	6	4	0	0	0	4	0
D 2	4	6	4	0	0	0	4	0
D 3	4	6	4	0	0	0	4	0
D 4	9	1	0	1	3	0	3	1
D 5	9	1	0	1	3	0	3	1
Totals	30	20	12	2	6	0	18	2

$$\mathrm{EG}(\textit{District Plan}) = \frac{18 - 2}{50} = 32\%$$



Red Gerrymander

Overall Percentages Seats		Overall Percentages Votes	
B	R	B	R
40	60	60	40

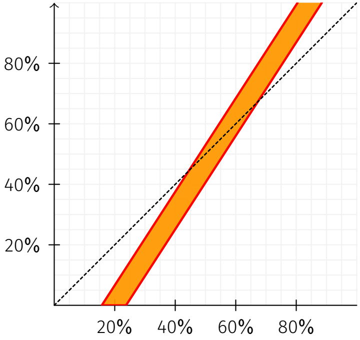
$$\text{sEG}(Blue) = (40 - 50) - 2(60 - 50) = -30\%$$

$$\text{sEG}(Red) = (60 - 50) - 2(40 - 50) = +30\%$$

In larger (two party) elections it's possible to approximate the Efficiency Gap for a party using the simplified Efficiency Gap for a Party:

$$\begin{aligned} \mathrm{EG}(\textit{District Plan}) &\approx |\,\mathrm{sEG}(\textit{Blue})| \\ &= |\,\mathrm{sEG}(\textit{Red}) \\ &= \frac{|\,\mathrm{sEG}(\textit{Party}\ 1)| + |\,\mathrm{sEG}(\textit{Party}\ 2)| + \cdots + |\,\mathrm{sEG}(\textit{Party}\ N)|}{N} \\ &\qquad \text{(with more than 2 parties)} \end{aligned}$$

The recommended range for the Efficiency Gap score is $\pm 8\%$, plotted on the axes below in orange. Proportional Representation occurs on the dotted black line.



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- False positives **and** false negatives

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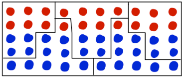
- False positives **and** false negatives
- Incompatible with Proportional Representation

Example

In Tiny State, overall voter preference is 60% Blue.
In the Red Gerrymander, 40% of the votes are for Red, who wins 60% of the seats.



If individual voters change preferences so that 40% of the overall preference is Blue, then there is partisan symmetry if the same districting plan now gives Blue 60% of the seats in most of the scenarios where Blue gets 40% of the vote. But...

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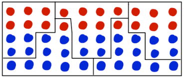

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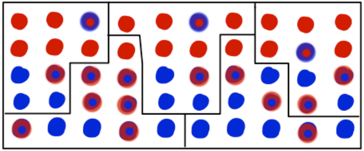

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PARTISAN SYMMETRY: DRAWBACKS

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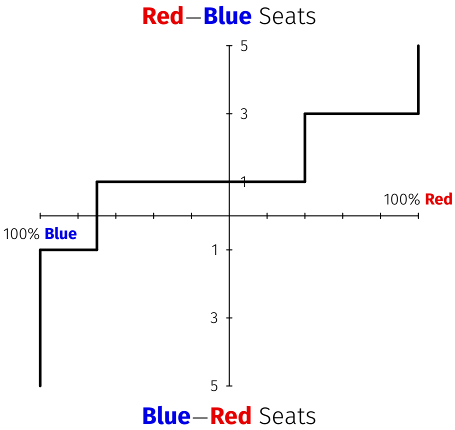
PARTISAN SYMMETRY: DRAWBACKS

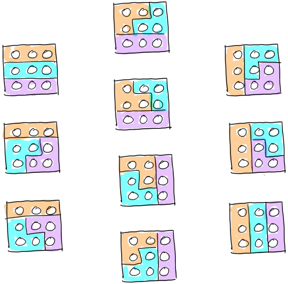
- A criterion for fairness without an associated metric (yet)
- Only for partisan gerrymandering
- More difficult to understand than a single number:



SAMPLING

- Use parallel processing algorithm to create the space of all districting plans (subject to constraints).

District Plans for Really Tiny State



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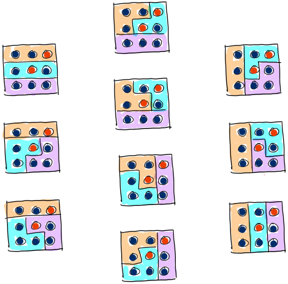

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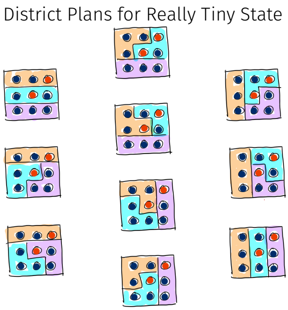
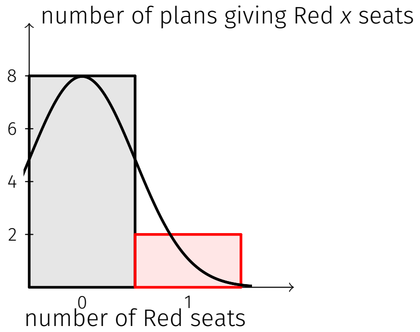
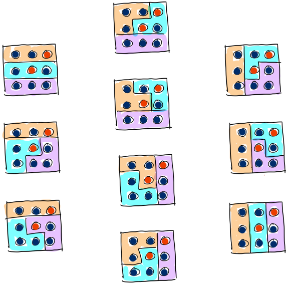
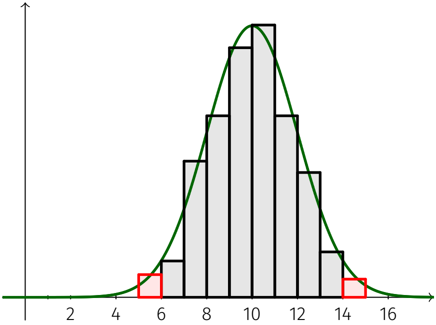




- Use parallel processing algorithm to create the space of all districting plans (subject to constraints).
- Feed in data to determine outcomes in each plan.
- Central Limit Theorem: with enough districting plans, the outcome data will be normally distributed – statistically detectable outliers!

District Plans for Really Tiny State





Hypotheses like racial gerrymandering, socioeconomic gerrymandering, can be tested in the space of all possible districts as long as there is appropriate census data.

Instead of trying to use one number to rank a districting plan in the abstract, we can rank a district plan's likelihood to have occurred randomly.

WHO’S WHO

Tufts Gerrymandering/Convex Geometry: Mira Bernstein & Moon Duchin, http://sites.tufts.edu/gerrymandr/

Software for Sampling: Princeton https://imai.princeton.edu/research/files/redist.pdf

Generally interesting political math: AMS Capital Currents Blog (esp. Karen Saxe) https://blogs.ams.org/capitalcurrents/author/ksaxe/

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5. Autoredistrict. http://autoredistrict.org/
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7. Fifield, Benjamin; Higgins, Michael; Imai, Kosuke; Tarr, Alexander. *A New Automated Redistricting Simulator Using Markov Chain Monte Carlo.* Preprint: https://imai.princeton.edu/research/files/redist.pdf